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Mechanical Stress Distribution in Functionally Graded Material's Artificial Hip Joints Implants Using Mathematical Model

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Abstract: Problems of the Total hip joint replacements leads the distribution of mechanical stresses over the implant-joint and bone. It was come to know during different experiments that using different combinations of load, Materials bonds and implants failure can be minimize. In this study FGM is used as implant material and using power low the mathematical model is proposed for mechanical stress distribution.

Keywords: *Mathematical molding, FEM, Total Hip Replacement, Artificial implants, FGM*

I. Introduction

Total hip replacement or prosthesis is the processer to replacing the hip joint with an artificial implant. Form the clinical point of view mechanical loading plays a vital role in strengthen the bone and enhance bone healing process [1].

There are different problems in traditional material like less rigidity, corrosion, metal incompatibility, shrinkage in less temperature, frequent breakdown and less life of joint etc. Moreover the stress developed in cement used is about 30-55% in intact cement-implant [8]. Functionally Graded material is proposed to avoid above problems, even though Cemented implants has good success rate, even though there are many cases of artificial joint failure or loosening prosthesis [2].

Multiple studies done on different bone cement by applying the multiple combination of mechanical loading, bone stiffness and weight of patient [18]. Failure of prosthesis is due to stresses generated by load distribution between bone and artificial implants (Turner (2005, [9, 11]) and loosening can be due to fatigue stress developed on implants day today routine activity [12, 13]

Functionally graded materials (FGM) are a remarkable class of composites that have a gradual variation of material properties from one surface to another. In this study FGM is used as implant material and using power low the mathematical model is proposed for mechanical stress distribution.

II. Mathmatical Model

A. Mechanical Stress Distribution

The following governing equations using power law can be generated and used for 2D stress on the artificial hip joint assembly.

$$\{(\rho_t-\rho_b)(\frac{z}{h}+\frac{1}{2})^n\}\frac{\partial^2}{\partial t}u-(\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y})=p_{11} \quad \ldots\ldots \text{(i)}$$

$$\{(\rho_t-\rho_b)(\frac{z}{h}+\frac{1}{2})^n\}\frac{\partial^2}{\partial t}v-(\frac{\partial\sigma_{12}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y})=p_{22} \quad \ldots\ldots \text{(ii)}$$

Equation of strain in another direction is

$$\varepsilon_{11}=\frac{\partial u}{\partial x} \quad \ldots\ldots \text{(iii)}$$

$$\varepsilon_{22}=\frac{\partial v}{\partial y} \quad \ldots\ldots \text{(iv)}$$

$$2\varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \quad \ldots\ldots \text{(v)}$$

Using generalized Hook's law

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}=\begin{bmatrix}(C_{11t}-C_{11b})(\frac{z}{h}+\frac{1}{2})^n+C_{11b}) & (C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b}) & 0\\ (C_{21t}-C_{21b})(\frac{z}{h}+\frac{1}{2})^n+C_{21b}) & (C_{22t}-C_{22b})(\frac{z}{h}+\frac{1}{2})^n+C_{22b}) & 0\\ 0 & 0 & (C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{bmatrix} \ldots\ldots \text{(vi)}$$

Here σ_{ij} is stress tensor in different direction, ε_{ij} is strain tensor, p_{ij} used for body forces, u and v are the displacement in x and y direction. C_{ij} are stiffness component, ρ is density

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Using the equations (iii) to (v), equation (i) and (ii) can be rewritten as

$$-\frac{\partial}{\partial x}\Big((C_{11t}-C_{11b})(\frac{z}{h}+\frac{1}{2})^n+C_{11b})\frac{\partial u}{\partial x}+(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial v}{\partial y}\Big)-\frac{\partial}{\partial y}\Big[(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\Big)\Big]=p_{11}-\rho\frac{\partial^2}{\partial t^2}u \quad \text{(vii)}$$

$$-\frac{\partial}{\partial x}\Big((C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial u}{\partial y}+(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial v}{\partial x}\Big)-\frac{\partial}{\partial y}\Big[(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial u}{\partial x}+(C_{22t}-C_{22b})(\frac{z}{h}+\frac{1}{2})^n+C_{22b})\frac{\partial v}{\partial y}\Big]=p_{22}-\rho\frac{\partial^2}{\partial t^2}v \ \ldots\ldots \text{(viii)}$$

Weak form using weight function W_1

$$\int W_1\Big[-\frac{\partial}{\partial x}\Big((C_{11t}-C_{11b})(\frac{z}{h}+\frac{1}{2})^n+C_{11b})\frac{\partial u}{\partial x}+(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n\frac{\partial v}{\partial y}\Big)-\frac{\partial}{\partial y}\Big\{(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\Big)\Big\}-p_{11}+\rho\frac{\partial^2}{\partial t^2}u\Big]dx\,dy=\oint W_1\left[\begin{matrix}\begin{pmatrix}(C_{11t}-C_{11b})(\frac{z}{h}+\frac{1}{2})^n+C_{11b})\frac{\partial u}{\partial x}+\\ (C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial v}{\partial y}\end{pmatrix}n_x+\\ n_y\Big\{(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\Big)\Big\}\end{matrix}\right]ds$$

…… (ix)

Rewrite Equation (ix)

$$\int\Big[-\frac{\partial W_1}{\partial x}\Big((C_{11t}-C_{11b})(\frac{z}{h}+\frac{1}{2})^n+C_{11b})\frac{\partial u}{\partial x}+(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial v}{\partial y}\Big)-\frac{\partial W_1}{\partial y}\Big[(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\Big)\Big]-W_1p_{11}+\rho W_1\frac{\partial^2}{\partial t^2}u\Big]dx\,dy-\oint\Big[W_1\Big((C_{11t}-C_{11b})(\frac{z}{h}+\frac{1}{2})^n+C_{11b})\frac{\partial u}{\partial x}+(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial v}{\partial y}\Big)n_x+n_yW_1\Big\{(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\Big)\Big\}\Big]ds=0 \quad \ldots\ldots\text{(x)}$$

Using weight function W_2

$$\int W_2\Big[-\frac{\partial}{\partial x}\Big((C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial u}{\partial y}+(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial v}{\partial x}\Big)-\frac{\partial}{\partial y}\Big\{(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial u}{\partial x}+(C_{22t}-C_{22b})(\frac{z}{h}+\frac{1}{2})^n+C_{22b})\frac{\partial v}{\partial y}\Big\}-p_{22}\,\rho\frac{\partial^2}{\partial t^2}v\Big]dx\,dy=\oint W_2\left[\begin{matrix}\Big((C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial u}{\partial y}+(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial v}{\partial x}\Big)n_x+\\ n_y\Big\{(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial u}{\partial x}+(C_{22t}-C_{22b})(\frac{z}{h}+\frac{1}{2})^n+C_{22b})\frac{\partial v}{\partial y}\Big\}\end{matrix}\right]ds$$

…… (xi)

Rewrite Equation (xi)

$$\int\Big[-\frac{\partial W_2}{\partial x}\Big((C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial u}{\partial y}+(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial v}{\partial x}\Big)-\frac{\partial W_2}{\partial y}\Big\{(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial u}{\partial x}+(C_{22t}-C_{22b})(\frac{z}{h}+\frac{1}{2})^n+C_{22b})\frac{\partial v}{\partial y}\Big\}-W_2p_{22}+W_2\,\rho\frac{\partial^2}{\partial t^2}v\Big]dx\,dy-\oint W_2\left[\begin{matrix}\Big((C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial u}{\partial y}+(C_{33t}-C_{33b})(\frac{z}{h}+\frac{1}{2})^n+C_{33b})\frac{\partial v}{\partial x}\Big)n_x\\ +n_y\begin{Bmatrix}(C_{12t}-C_{12b})(\frac{z}{h}+\frac{1}{2})^n+C_{12b})\frac{\partial u}{\partial x}+\\ (C_{22t}-C_{22b})(\frac{z}{h}+\frac{1}{2})^n+C_{22b})\frac{\partial v}{\partial y}\end{Bmatrix}\end{matrix}\right]ds=0$$

…… (xii)

Displacement u and v in term of interpolation functions given as

$$u(x,y,t)=\sum_{i=1}^{n}N_i(x,y,t)\,u_i(t) \quad \ldots\ldots \text{(xiii)}$$

$$v\,(x,y,t)=\sum_{i=1}^{n}N_i(x,y,t)\,v_i(t) \quad \ldots\ldots\text{(xiv)}$$

$$Wi(x,y,t)=\sum_{i=1}^{n}N_i(x,y,t)\,W_i(t) \quad \ldots\ldots \text{(xv)}$$

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Put the equations (xiv) - (xv) in equations (x) and (xii) and write the final equations in matrix form.

Mü+A(u)u=P ……(xvi)

Equation (xix) can be solved by any suitable solving method like MOL, GDM, DEM, and QNM.

B. Heat Transfer

The heat transfer can be analyze using unsteady heat equation:

$$\rho(H_{11t} - H_{11b})(\frac{z}{h} + \frac{1}{2})^n + H_{11b})\frac{\partial\theta}{\partial t} + \nabla.(-\alpha\nabla\theta) = 0 \text{…… (xx)}$$

Here, θ used for temperature, H is heat capacity, α is thermal conductivity, t is for time.

Naviour-stoke and Brinkman equations use for the cemented flow

$$(\rho_{11t} - \rho_{11b})(\frac{z}{h} + \frac{1}{2})^n + \rho_{11b})\frac{\partial u}{\partial t} - \nabla.\tau_m + (\rho_{11t} - \rho_{11b})(\frac{z}{h} + \frac{1}{2})^n + \rho_{11b})(u.\nabla)u + \nabla p_m = p \quad \text{…… (xxi)}$$

Where u is velocity of fluid, p_m is the pressure, p represents force, τ_m shear tensor, D is deformation tensor

To model flow in porous media Brinkman equation used

$$(\rho_{11t} - \rho_{11b})(\frac{z}{h} + \frac{1}{2})^n + \rho_{11b})\frac{\partial v}{\partial t} - \nabla.\tau_c + \frac{\eta}{k}v + \nabla p_c = p \text{ …… (xxii)}$$

The (η) is viscosity and (Υ) is shear rate

III. CONCLUSIONS

Governing differential equations can give a good result with appropriate boundary conditions. Above generated differential equation can be solved any above mention technique and distribution/deformation of stresses can found. Impact of some more parameter still not included in the present study.

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